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It may be remarked that  $d$  is a mean proportional between  $a$  and  $m$ ,  $l$  is a mean proportional between  $3a$  and  $m$ , and  $l$  is the diagonal of a cube whose face-diagonal is  $s$  and edge  $d$ .

(6). To determine a point equidistant from all the vertices.

$LK$  bisects  $MH$  at  $O$ .

In the rectangle of which  $GH$  and  $MN$  are opposite sides,  $GN$  bisects  $MH$ , therefore passes through  $O$  and is itself bisected at  $O$ .

So with all the longest diagonals.

Hence  $O$  is equidistant from all the vertices.

COROLLARY 1.  $O$  is equidistant from all the edges.

COROLLARY 2.  $O$  is equidistant from all the faces.

(7). To compute the volume.

This could be done by conceiving the dodecahedron composed of twelve equal pyramids with  $O$  as their common vertex and the faces for bases. But another method will be adopted.

The dodecahedron is composed of a cube one of whose faces is  $EFGC$  and six equal truncated triangular prisms, one of which has  $AB$ ,  $FG$ , and  $EC$  for its lateral edges. A right section of this truncated prism is a triangle with base equal to  $EF$ , and altitude, the perpendicular from  $A$  to  $UV$ . Since  $ABXY$  is a rectangle equal to  $LMKH$ , this perpendicular is one-half the difference between  $AX (= LH)$  and an edge of the cube, and, therefore, equals  $\frac{1}{2}a$ .

Hence the area of the right section is  $\frac{a^2}{8}[\sqrt{5}+1]$ , and the volume of the truncated prism is  $\frac{a^2}{8}[\sqrt{5}+1]\frac{a+a[\sqrt{5}+1]}{3}$ .

The sum of the volumes of six such solids is  $\frac{a^3}{4}[7+3\sqrt{5}]$ .

$$\text{Volume of cube} = \frac{a^3}{4}[8+4\sqrt{5}]$$

$$\text{Volume of dodecahedron} = \frac{a^3}{4}[15+7\sqrt{5}]$$

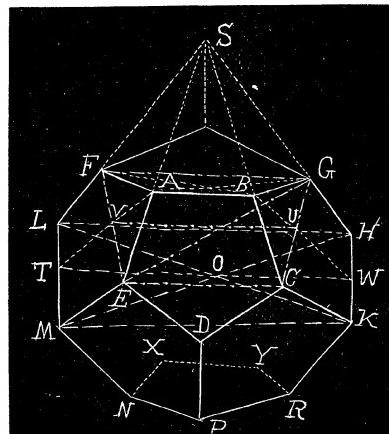
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134. Proposed by J. C. GREGG, A. M., Superintendent of Schools, Brazil, Ind.

If  $ABCD$  is a quadrilateral circumscribing a circle, show that the line joining the middle points of the diagonals  $AB$ ,  $CD$  passes through the center of the circle.

Solution by HARRY S. VANDIVER, Bala, Penna.

We will use quadrilinear coördinates denoting the equation of the four



sides  $\alpha=0$ ,  $\beta=0$ ,  $\gamma=0$ ,  $\delta=0$ , and the *lengths* of the corresponding sides by  $a$ ,  $b$ ,  $c$ , and  $d$ . Let radius of circle be  $r$ . Then the equation of the line joining the middle points of the diagonals is  $a\alpha-b\beta+c\gamma-d\delta=0 \dots (1)$  (Cf. Salmon's *Conics*, page 54, Ex. 5).

Putting  $\alpha=\beta=\gamma=\delta=r$  we obtain  $r(a-b+c-d)=0$ , which is satisfied since  $a+c=b+d$ .

**137.** Proposed by J. W. YOUNG, Oliver Graduate Scholar in Mathematics, Cornell University, Ithaca, N. Y.

A right cone has its vertex in a horizontal plane, its axis being perpendicular to the plane. A string has one extremity attached to a point on the cone. The other extremity,  $P$ , of the string is kept in the plane, and the string is then wound around the cone, without being allowed to slip. Show that the spiral generated by  $P$  cuts all straight lines through the vertex at the same angle.

Solution by the PROPOSER.

Let  $P$ ,  $P'$  be two points on the spiral;  $Q$ ,  $Q'$  the corresponding points in the path of the string around the cone;  $N$ ,  $N'$  the points where the perpendiculars from  $Q$ ,  $Q'$  to the plane through the vertex  $O$  of the cone, cut the plane.

The right-angled triangles  $QNO$ ,  $Q'N'O$  have the angles  $QON$  and  $Q'ON'$  equal; hence they are similar.

$$\therefore \frac{QN}{ON} = \frac{Q'N'}{ON'} \dots (1).$$

Again, since the string must not slip, it makes a constant angle with the plane.

$\therefore \triangle QNP$  is similar to  $\triangle Q'N'P'$ .

$$\therefore \frac{PN}{QN} = \frac{P'N'}{Q'N'} \dots (2).$$

From (1) and (2),

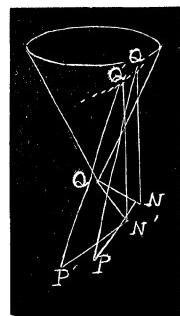
$$\therefore \frac{PN}{ON} = \frac{P'N'}{ON'} \dots (3).$$

But the triangles  $ONP$  and  $ON'P'$  are right-angled at  $N$  and  $N'$  ( $PN$ ,  $P'N'$  being the projections to tangents to the circular cone). From (3),

$\therefore \triangle ONP$  is similar to  $\triangle ON'P'$ .

$$\therefore \angle OPN = \angle OP'N'.$$

Observing that  $PN$  and  $P'N'$  are normals to the spiral, the last equation states that the normals make a constant angle with rays through  $O$ . Q. E. D.



#### AVERAGE AND PROBABILITY.

**90.** Proposed by WALTER H. DRANE, Graduate Student, Harvard University, Cambridge, Mass.

During a heavy rain storm a circular pond is formed in a circular field. If a man undertakes to cross the field in the dark, what is the chance that he will walk into the pond? [From *Byerly's Integral Calculus*.]

I. Solution by B. F. FINKEL, A. M., M. Sc., Professor of Mathematics and Physics, Drury College, Springfield, Mo.

Let  $O$  be the center of the circular field, and  $R$  its radius;  $C$  the center of